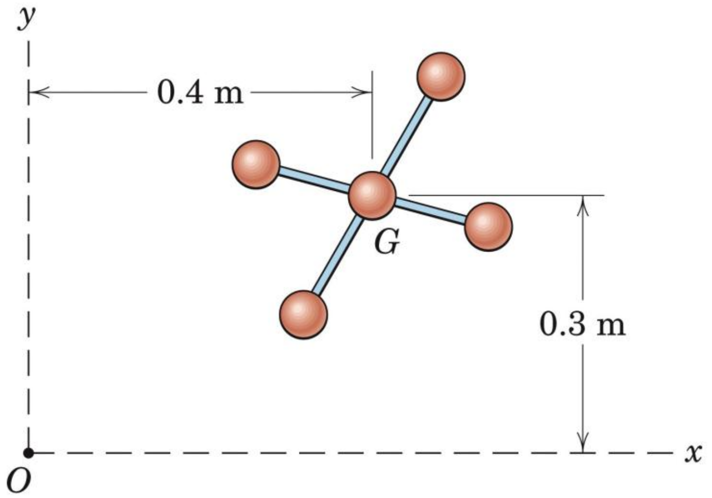
ÇANKAYA UNIVERSITY – MECHANICAL ENGINEERING DEPARTMENT
ME 204 – DYNAMICS – SPRING 2013
HOMEWORK 3
KINETICS OF SYSTEMS OF PARTICLES

Due Date: 3rd Lecture Hour of Week 8

PROBLEM 4/14

Each of the five connected particles has a mass of 0.6 kg, with G as the center of mass of the system. At a certain instant the angular momentum of the system about G is $1.20\mathbf{k}$ kg·m²/s, and the x- and y-components of the velocity of G are 3 m/s and 4 m/s, respectively. Calculate the angular momentum $\mathbf{H}_O$ of the system about O for this instant.



$$\underline{H}_O = \underline{H}_G + \underline{\bar{r}} \times \underline{G} \quad , \quad \underline{G} = 3(3\underline{i} + 4\underline{j}) \text{ kg·m/s}$$

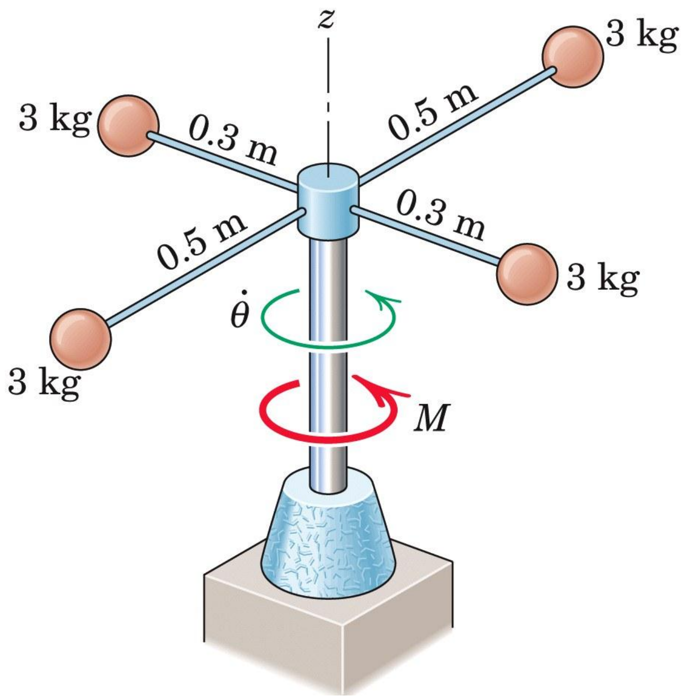
$$= 1.20\underline{k} + (0.4\underline{i} + 0.3\underline{j}) \times 3(3\underline{i} + 4\underline{j})$$

$$= 1.20\underline{k} + 3(1.6\underline{k} - 0.9\underline{k})$$

$$= 1.20\underline{k} + 3(0.7\underline{k}) = \underline{3.3\underline{k} \text{ kg·m}^2/\text{s}}$$

PROBLEM 4/18

The four 3-kg balls are rigidly mounted to the rotating frame and shaft, which are initially rotating freely about the vertical z-axis at the angular rate of 20 rad/s clockwise when viewed from above. If a constant torque $M = 30$ N·m is applied to the shaft, calculate the time t to reverse the direction of rotation and reach an angular velocity $\dot{\theta} = 20$ rad/s in the same sense as M.



$$\int_0^t M_z\,dt = H_{z_2} - H_{z_1} \quad , \quad H_z = \sum m_i r_i (r_i \dot{\theta})$$

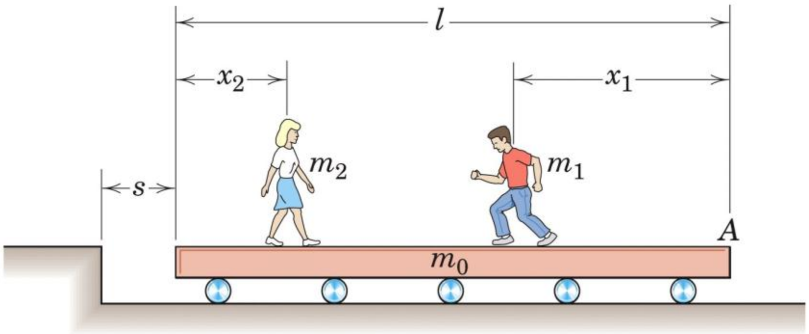
$$H_z = 2(3)(0.3)^2\dot{\theta} + 2(3)(0.5)^2\dot{\theta} = 2.04\dot{\theta}$$

so $30t = 2.04(20 - [-20]) = 81.6$

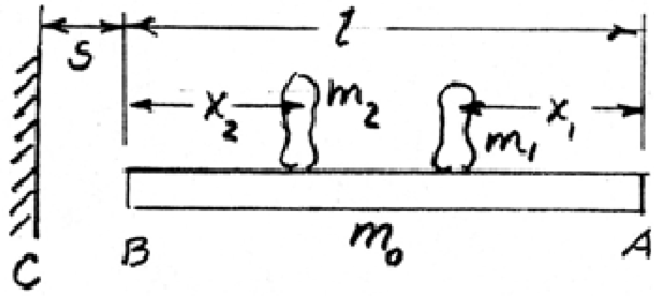
$\underline{t = 2.72 \text{ s}}$

PROBLEM 4/22

The man of mass m_1 and the woman of mass m_2 are standing on opposite ends of the platform of mass m_0 which moves with negligible friction and is initially at rest with $s = 0$. The man and woman begin to approach each other. Derive an expression for the displacement s of the platform when the two meet in terms of the displacement x_1 of the man relative to the platform.



With respect to C, $\Sigma m_i x_i = \text{constant}$



$$m_1 l + m_2(0) + m_0 \frac{l}{2} =$$
$$m_1(s + l - x_1) + m_2(s + x_2) + m_0\left(s + \frac{l}{2}\right)$$

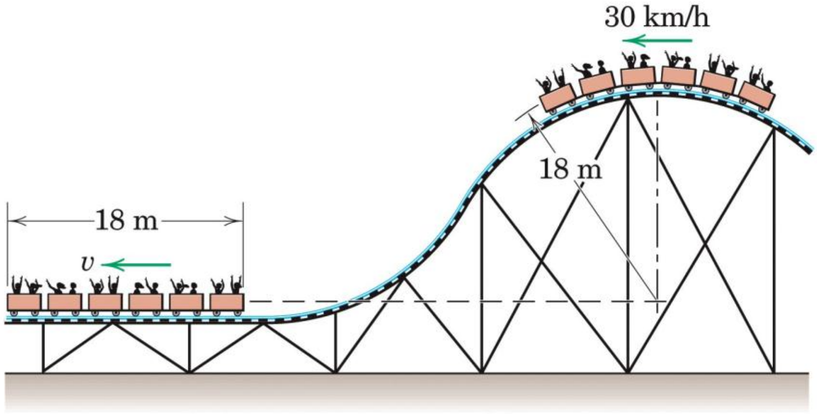
Simplify & get $s = \dfrac{m_1 x_1 - m_2 x_2}{m_0 + m_1 + m_2}$

But they meet when $x_2 + x_1 = l$ so

$$s = \frac{(m_1 + m_2)\,x_1 - m_2 l}{m_0 + m_1 + m_2}$$

PROBLEM 4/29

The cars of a roller-coaster ride have a speed of 30 km/h as they pass over the top of the circular track. Neglect any friction and calculate their speed v when they reach the horizontal bottom position. At the top position, the radius of the circular path of their mass centers is 18 m, and all six cars have the same mass.



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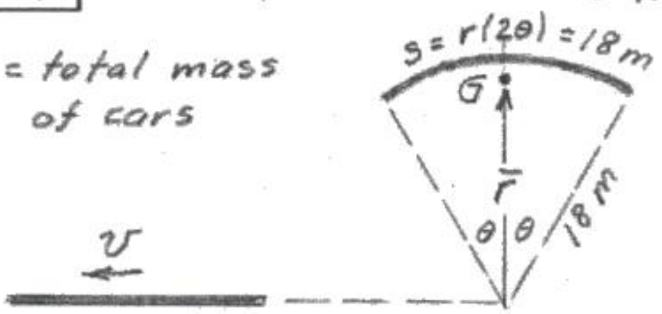
m = total mass of cars



$$\theta = \frac{s}{2r} = \frac{18}{2(18)} = \frac{1}{2}\text{ rad}$$

$$= \frac{1}{2}\,\frac{180}{\pi} = 28.65^\circ$$

$$\bar{r} = \frac{r\sin\theta}{\theta} = \frac{18\sin 28.65^\circ}{1/2} = 17.26\text{ m}$$

For system $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}m\left(v^2 - \left[\frac{30}{3.6}\right]^2\right) - mg(17.26) = 0$$

$$v^2 = (30/3.6)^2 + 2(9.81)(17.26) = 69.4 + 338.6$$

$$= 408\ (\text{m/s})^2$$

$v = 20.2$ m/s or $v = 20.2(3.6) = \underline{72.7\text{ km/h}}$